Abstracts

Keynote speakers

Tree decompositions meet induced matchings

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Joint work with: Tara Abrishami, Marcin Briański, Jadwiga Czyżewska, Rose McCarty, Paweł Rzążewski, and Bartosz Walczak

Tree decompositions are an important tool in structural and algorithmic graph theory, largely due to treewidth. Several more general width parameters have been recently introduced that, unlike treewidth, can also be bounded on dense graph classes, and for which Maximum Independent Set and related problems can be solved in polynomial time when the parameter is bounded. This includes tree-independence number, introduced independently by Yolov in 2018 and by Dallard, Milanič, and Štorgel in 2021, and induced matching treewidth, introduced by Yolov in 2018 (under the name minor-matching hypertreewidth). Good algorithmic properties of these parameters motivate the study of conditions for their boundedness.

The induced matching treewidth of a graph G is defined as the minimum, over all tree decompositions of G, of the maximum size of an induced matching all of whose edges intersect the same bag of the decomposition. We discuss structural properties of graphs with bounded induced matching treewidth. First, we show that graphs with bounded induced matching treewidth that exclude a fixed biclique as an induced subgraph admit a tree decomposition in which no bag contains a large independent set. Second, we show that classes of graphs with bounded induced matching treewidth and bounded clique number have bounded

chromatic number. These results prove two recent conjectures of Lima, Milanič, Muršič, Okrasa, Rzążewski, and Štorgel (ESA 2024).

Separation systems of subdivisions

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Joint work with: George Kontogeorgiou, Matías Pavez-Signé, S. Taruni and Ana Trujillo-Negrete

A (strong) separating path system of a graph G is a family of paths such that for each ordered pair of edges of G there is a path which contains the first edge but does not contain the second edge. There has been a surge of interest in finding small separating path systems in the last decade. Bonamy et al. showed that the smallest size of a separating path system of an n-vertex graph G is O(n) which is best possible by results of Balogh et al.

Recently, Botler and Naia conjectured that we could replace the paths that form the separating system with other families of graphs. Namely, they conjectured that for any fixed graph H with at least one edge, any n-vertex graph G has a separating system of size O(n) consisting of subdivisions of H and single edges. (The single edges are necessary, and we allow dependence on H in the term O(n).)

We show Botler and Naia's conjecture is true.

Trigraphs and dense neighborhood lemmas

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Joint work with: Romain Bourneuf and Pierre Charbit

Trigraphs are graphs in which some edges are colored red, usually to highlight the fact that they can be both edges and non edges. The aim of this talk is to show how they can bridge the integrality gap in combinatorial optimization. For instance, in a trigraph T, the computation of the fractional domination number is done without using red edges whereas the domination number is computed using both normal and red edges. Hence the gap is smaller than in the usual setting where we would only dominate by normal edges.

To illustrate, consider a subset V of points in \mathbb{R}^N such that every ball B(v,1) with $v \in V$ intersects V on at least $\delta |V|$ elements. In general, there is no bounded (in terms of δ) cover of V by balls B(v,1), this is only true in bounded dimension. However, for every $\varepsilon>0$, a $f(1/\delta,1/\varepsilon)$ cover of V by balls $B(v,1+\varepsilon)$ exists. A graph interpretation helps: the threshold graph G on vertex set V and edges xy when $d(x,y)\leq 1$ has fractional domination $O(1/\delta)$ but unbounded domination. However, when adding red edges with length in $]1,1+\varepsilon]$, the resulting trigraph T has bounded domination. The reason is that T has bounded VC-dimension whereas G has not.

The previous statement is an example of a dense neighborhood lemma (DNL): we are given two types of neighborhoods, a small one (balls of radius 1) and a large one (balls of radius $1+\varepsilon$), and DNL asserts that the cover using large neighborhoods is function of the fractional cover using the small ones. There are numerous examples of such statements, which are not always based on metric spaces. A clustering version of DNL, in which V is partitioned into similar points, provides a Regularity Lemma type partition with an exponential number of parts. It sits between the ultra-strong version (polynomial number of parts in bounded VC-dimension) and the usual one (tower function number of parts).

Some applications: every triangle-free graph with minimum degree $n/3-n^{1-\varepsilon}$ has bounded chromatic number (this does not hold with $n/3-n^{1-o(1)}$).

Every $(0.7+\varepsilon)n$ -regular K_5 -free graph has bounded chromatic number (and DNL gives all chromatic thresholds for regular K_t -free graphs). In tournaments, DNL implies that the domination number is bounded by some function of the fractional (acyclic) chromatic number. Finally, every $(1/2-\varepsilon)$ -majority digraph arising from a voting process has bounded domination

Further invited speakers

The Pairing-Hamiltonian property in graph prisms

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Joint work with: G. Mazzuoccolo, F. Romaniello, J.P. Zerafa

Let G be a graph of even order, and consider K_G as the complete graph on the same vertex set as G. A perfect matching of K_G is called a pairing of G. If for every pairing M of G it is possible to find a perfect matching N of G such that $M \cup N$ is a Hamiltonian cycle of K_G , then G is said to have the Pairing-Hamiltonian property, or PH-property, for short. In 2007, Fink $[J.\ Combin.\ Theory\ Ser.\ B,\ \bf 97]$ proved that for every $d \geq 2$, the d-dimensional hypercube \mathcal{Q}_d has the PH-property, thus proving a conjecture posed by Kreweras in 1996.

In this talk we will see how Fink's result can be extended to proving that given a graph G having the PH-property, the prism graph $\mathcal{P}(G)$ of G has the PH-property as well. Moreover, if G is a connected graph, we will see that there exists a positive integer k_0 such that the k^{th} -prism of a graph $\mathcal{P}^k(G)$ has the PH-property for all $k \geq k_0$.

Generating maps the naïve way

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There are several algorithms – often even implemented as programs – for the generation of various classes of plane graphs. For higher surfaces only very few algorithms exist, which is on one hand due to the fact that of course genus 0 is an especially interesting case, but also due to the fact that without the Jordan Curve Theorem the constructions get increasingly complicated. In this talk we present a naïve algorithm that is very general and nevertheless sufficiently fast in many cases.

On Gyárfás' Path-Colour Problem

Ben Cameron

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Joint work with: Alexander Clow (SFU)

Gyárfás posed the following conjecture in 1997: there exists a constant k such that if each path of a graph spans a 3-colourable subgraph, then the graph is k-colourable. It is noted that k=4 might suffice. Let r(G) be the maximum chromatic number of any subgraph H of G where H is spanned by a path.

In this talk, we sketch our constructive proof that for all $r \geq 6$, there exists a graph G with $r(G) \leq r$ and $\chi(G) \geq \lfloor \frac{3r}{2} \rfloor -1$. Hence, for all constants k there exists a graph with $\chi-r>k$. If time permits, we will also discuss some preliminary progress on comparing $\chi(G)$ and r(G) for all H-free graphs G for various graphs H.

200,000 colors suffice

(for t-perfect graphs)

Linda Cook

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Joint work with: Maria Chudnovsky, James Davies, Sang-il Oum, Jane Tan

Perfect graphs can be described as the graphs whose stable set polytopes are defined by their non-negativity and clique inequalities. In 1975, V. Chvátal defined an analogous class called t-perfect graphs, which are the graphs whose stable set polytopes are defined by their non-negativity, edge inequalities, and odd circuit inequalities. We show that t-perfect graphs are 200,000-colourable. This is the first finite bound on the chromatic number of t-perfect graphs, and answers a question of B. Shepherd from 1995.

This bound is probably not tight; M. Laurent and P. Seymour gave an example of a t-perfect graph requiring four colors in the 1990's and it is open whether all t-perfect graphs are 4-colorable. Our proof is mainly graph theoretic and uses techniques developed in the context of " χ -boundedness". In this talk we discuss this result and related open problems.

3-Colouring Planar Graphs

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Joint work with: Pat Morin, Sergey Norin, David R. Wood

We show that every n-vertex planar graph is 3-colourable with monochromatic components of size $O(n^{4/9})$. The best previous bound was $O(n^{1/2})$ due to Linial, Matoušek, Sheffet and Tardos [Combin. Probab. Comput., 2008].

Periodic colorings and orientations

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Joint work with: Tara Abrishami, Ugo Giocanti, Matthias Hamann, Paul Knappe, and Rögnvaldur G. Möller)

A coloring or orientation of an infinite graph G is periodic if V(G) has finitely many orbits under the action of the group of automorphisms of G preserving the coloring or the orientation. A necessary condition for having a periodic coloring or orientation is that G must be quasi-transitive, that is V(G) must have finitely many orbits under the action of the automorphism group of G.

We present examples of quasi-transitive graphs of treewidth 2 with no periodic colorings or orientations, providing strong negative answers to problems of Bowen and Lyons, and of Esperet, Giocanti and Legrand-Duchesne. On the other hand, we show that every quasi-transitive graph G of bounded pathwidth has a periodic coloring with $\chi(G)$ colors.

Infinite induced-saturated graphs

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Joint work with: Marthe Bonamy, Tom Johnston, Tash Morrison and Alex Scott

We call a graph H-free if it does not have H as an induced subgraph. We show that for every finite graph H which is not a clique or a stable set, there is a countable H-free graph G such that any graph G' obtained from G by adding or removing edges in a "locally finite" manner does contain H as an induced subgraph.

Complete polyhedral description of chemical graphs of maximum degree at most 3

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Joint work with: Valentin Dusollier, Sébastien Bonte, Gauvain Devillez, Hadrien Mélot, David Schindl

Chemical graphs are simple undirected connected graphs, where vertices represent atoms in a molecule and edges represent chemical bonds. A degree-based

topological index is a molecular descriptor used to study specific physicochemical properties of molecules. Such an index is computed from the sum of the weights of the edges of a chemical graph, each edge having a weight defined by a formula that depends only on the degrees of its endpoints. Given any degree-based topological index and given two integers n and m, we are interested in determining chemical graphs of order n and size m that maximize or minimize the index. Focusing on chemical graphs with maximum degree at most 3, we show that this reduces to determining the extreme points of a polytope that contains at most 10 facets. We also show that the number of extreme points is at most 16, which means that for any given n and m, there are very few different classes of extremal graphs, independently of the chosen degree-based topological index.

Generation of vertex-girth-regular graphs

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Joint work with: Robert Jajcay, István Porupsánszki

This talk is centered around a problem that naturally bridges two important areas of graph theory: the cage problem and vertex-transitive graphs (see for example the survey [1]).

For integers v,k,g and λ , a vertex-girth-regular (v,k,g,λ) graph is a graph on v vertices in which every vertex has k neighbours, the length of the shortest cycle is equal to g and every vertex is contained in exactly λ shortest cycles. We consider the problem of determining the smallest such graphs for fixed k,g and λ . We discuss an algorithm that can effectively generate all vertex-girth-regular (v,k,g,λ) graphs for given integers v,k,g and λ and use this to determine the smallest such graphs for small parameters.

References

- [1] G. Exoo and R. Jajcay. Dynamic cage survey. *Electron. J. Combin.*, DS16:48, 2008.
- [2] R. Jajcay, J. Jooken and I. Porupsánszki. On vertex-girth-regular graphs: (Non-)existence, bounds and enumeration. *arXiv preprint arXiv:2408.14557*, 2024.

What is the fewest independent sets a triangle-free graph can have?

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We give a lower bound, in terms of the average degree d and the number n of vertices, in answer to the question in the title. As d grows, our bound is asymptotically sharp, up to several leading terms, as certified by an elementary random graph construction. Our proof relies on a deft induction technique due to Shearer (1983), together with some extra analysis.

Berge conjecture for generalised cycle permutation graphs

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Joint work with: Ján Karabáš, Roman Nedela, Martin Škoviera

The perfect matching index of a cubic graph G, denoted by pmi(G) is the smallest number of perfect matchings that cover all the edges of G. In 1970's, Berge conjectured that $pmi(G) \leq 5$ for every bridgeless cubic graph. If $pmi(G) \leq 4$, then G is not a counterexample to several deep and long-standing conjectures, the cycle double cover conjecture being among them.

A permutation graph is a cubic graph consisting of two cordless circuits connected by a perfect matching. Bridgeless cubic graphs that admit a 2-factor consisting of two disjoint circuits are called generalized permutation graphs. The minimal number of chords in a 2-factor with two circuits of a generalized permutation graph G is called permutation error of G, denoted by pe(G). Clearly, for a generalized permutation graph G we have Pe(G) = 0 whenever G is a permutation graph.

In 2009, Fouquet and Vanherpe proved that the perfect matching index of every permutation graph, with the sole exception of the Petersen graph, is at most 4. We generalize this result by showing that if $pe(G) \leq 1$, then $pmi(G) \leq 4$. The family of cubic graphs with $pe(G) \leq 1$ appears to be rich, for example, it includes the Isaacs flower snarks.

We also observe that it seems that the congruence phenomenon of the nonexistence of permutation snarks (that is cubic graphs with no 3-edge-colouring) on $6 \pmod 8$ vertices extends to generalized permutation snarks with permutation error at most 2.

Improved bounds for the minimum degree of minimal Ramsey graphs

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Joint work with: Yamaan Attwa, Tibor Szabó, and Jacques Verstraete

In 1976, Burr, Erdős and Lovász initiated a systematic study of various properties of Ramsey graphs. A graph G is said to be r-Ramsey if every r-colouring of the edges of G contains a monochromatic copy of K_k . Moreover, it is called minimal when no proper subgraph of G has this property. While historically a lot of attention has been devoted to the smallest possible order of a 2-Ramsey graph, one could also consider for example the smallest chromatic number, maximum, or minimum degree of a minimal r-Ramsey graph.

While Burr, Erdős and Lovász already determined the smallest possible minimum degree of minimal 2-Ramsey graphs, the behavior of this quantity is much less understood for more colours. In particular, it is not clear what the correct asymptotic behaviour (in both k and r) is. We will present some recent progress towards this problem.

Uniquely Normal Graphs

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Joint work with: Yilun Luo, Edita Máčajová

In this talk, we first give a brief review of normal edge-colourings of cubic graphs and their connection to the Petersen Colouring Conjecture. This famous conjecture is relevant because it implies other well-known conjectures, like the Cycle Double Cover and the Berge-Fulkerson conjectures.

A normal 5-edge-colouring of a cubic graph is a proper 5-edge-colouring where, for each edge, the set formed by the colour of the edge itself and the four edges incident to its endpoints contains either exactly five different colours (in which case the edge is called rich), or exactly three different colours (in which case the edge is called poor).

The main part of the talk is about a special family of graphs called uniquely normal graphs. These are graphs where every normal edge-colouring gives the same division of edges into rich and poor. We study some specific types of uniquely normal graphs and examine their structure and colouring properties. As an additional result, we use these graphs to construct an infinite family of counterexamples to a recent conjecture by Sedlar and Škrekovski.

Erdős-Pósa property of cycles that are far apart

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Joint work with: Vida Dujmović, Gwenaël Joret, Pat Morin

We prove that there exist functions $f,g:\mathbb{N}\to\mathbb{N}$ such that for all nonnegative integers k and d, for every graph G, either G contains k cycles such that vertices of different cycles have distance greater than d in G, or there exists a subset X of vertices of G with $|X|\leq f(k)$ such that $G-B_G(X,g(d))$ is a forest, where

 $B_G(X,r)$ denotes the set of vertices of G having distance at most r from a vertex of X.

Spanning plane subgraphs of 1-plane graphs

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Joint work with: Katsuhiro Ota, Yusuke Suzuki

A graph drawn on the plane is called 1-plane if each edge is crossed at most once by another edge. The problem of this talk is, for a given 1-plane graph G, to find a highly connected spanning subgraph of G that is a plane graph with respect to the drawing of G. We consider the condition of k and ℓ such that every k-connected 1-plane graph has an ℓ -connected spanning plane subgraph. Among others, we show that every 4-edge-connected 1-plane graph has a connected spanning plane subgraph and that there exist infinitely many 4-connected 1-plane graphs that have no 2-connected spanning plane subgraphs.

On linearly ordered colourings of hypergraphs

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Joint work with: B. Bedert, T.-V. Nakajima, and S. Živný

The classic notion of (vertex) graph colouring can be generalized to hypergraphs in several interesting ways; the most popular one avoids monochromatic hyperedges. In this talk, we focus on a lesser-known notion of *linearly ordered* k-colourings (LO_k -colourings) of hypergraphs.

An LO_k -coloring of a hypergraph H assigns colors $\{1,\ldots,k\}$ to the vertices of H in a way that in each hyperedge the maximum colour appears exactly once. Observe that the problem of LO_2 -colouring 3-uniform hypergraphs corresponds precisely to the classic NP-complete Positive-1-in-3-Sat problem (with vertices being interpreted as variables and hyperedges as clauses). This naturally leads to approximation question: for $\ell>k\geq 2$, given an LO_k -colourable hypergraph, find its LO_ℓ -colouring.

I will show that, if we restrict ourselves to 3-uniform hypergraphs, there exists $\epsilon>0$ such that in polynomial time we can LO_ℓ -colour an LO_2 -colourable graph for $\ell=(1-\epsilon)\log n$.

Spectral properties of hexagonal tilings of the torus

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Joint work with: Nino Bašić, Patrick W. Fowler, and Barry T. Pickup

Hexagonal tilings (honeycombs) of the torus are well-understood objects. It is therefore surprising that no comprehensive treatment of their spectra can be

found in the literature (at least to the best of our knowledge). The aim of this talk is to remedy this situation. We shall also discuss the graph energy of the underlying graphs and show that, as both radii of the torus tend to infinity, the graph energy per vertex converges to a constant known in theoretical chemistry circles as the *graphene bond number*. Moreover, we shall present an interesting connection between this constant and a random walk problem, which enables us to provide an explicit formula for the graphene bond number in terms of the gamma function.

Burling graphs and where to find them

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Joint work with: T. Abrishami, M. Briański, J. Davies, X. Du, J. Masaříková, B. Walczak

In 1965 Burling constructed a sequence of triangle-free graphs B_k with $\chi(B_k)=k$. A Burling graph is an induced subgraph of some B_k . In recent years Burling graphs attracted significant attention as they appeared to provide a negative answer to several open questions. For example, Erdős asked if intersection graphs of segments are χ -bounded. This is not the case, as Burling graphs are intersection graphs of segments, as shown by Pawlik, Kozik, Krawczyk, Lasoń, Micek, Trotter, and Walczak [JCTB 2014].

However, it turns out that much more is true. We prove that in broad classes of graphs, including intersection graphs of segments or strings, large chromatic number can be forced *only* by large cliques or graphs B_k , for large k. As a consequence, Burling graphs form a minimal hereditary class of graphs with unbounded

chromatic number — the second known class of graphs with this property after the class of complete graphs.

Cycle covers of cubic graphs with defect 3, and beyond

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Joint work with: Ján Karabáš, Edita Máčajová, Roman Nedela

A longstanding conjecture of Alon and Tarsi (1985), and independly of Jaeger (1985), suggests that the edges of every bridgeless graph can be covered with cycles of total length at most $7/5 \cdot m$, where m is the number of edges. The conjecture is closely related to several other problems in graph theory; in particular, it implies the celebrated cycle double cover conjecture. The 7/5-conjecture is particularly interesting for cubic graphs: it trivially holds for 3-edge-colourable cubic graphs, but very little is known about cycle covers of snarks, 2-connected cubic graphs that cannot be 3-edge-coloured. After 40 years, the 7/5-conjecture remains widely open.

We study the relationship between cycle covers and structural properties of snarks, focusing on their colouring defect. This invariant, introduced by Steffen in 2015, is defined as the minimum number of edges left uncovered by any set of three perfect matchings. We show that every bridgeless cubic graph with colouring defect not exceeding 3 admits a cycle cover of length at most $4/3 \cdot m + 1$, just one step above the universal lower bound of $4/3 \cdot m$ for all cubic graphs. We also prove that, regardless of defect, the same bound holds for bridgeless cubic graphs

that have an edge whose endvertices removed yield a 3-edge-colourable graph and the edge lies on a 5-cycle.

Fractional chromatic number vs. Hall ratio

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Given a graph G, its $Hall\ ratio\ \rho(G) = \max_{H\subseteq G} \frac{|V(H)|}{\alpha(H)}$ forms a natural lower bound for its fractional chromatic number $\chi_f(G)$. A recent line of research studied the question whether $\chi_f(G)$ can be bounded in terms of a (linear) function of $\rho(G)$. Dvořák, Ossona de Mendez and Wu [Combinatorica, 2020] gave a negative answer by proving the existence of graphs with bounded Hall ratio and arbitrarily large fractional chromatic number. In this paper, we solve two follow-up problems that were raised by Dvořák et al. The first problem concerns determining the growth of g(n), defined as the maximum ratio $\frac{\chi_f(G)}{\rho(G)}$ among all n-vertex graphs. Dvořák et al. obtained the bounds $\Omega(\log\log n) \leq g(n) \leq O(\log n)$. We show that the true value is close to the upper bound: $g(n) = (\log n)^{1-o(1)}$. The second problem posed by Dvořák et al. asks for the existence of graphs with bounded Hall ratio, arbitrarily large fractional chromatic number and such that every subgraph contains an independent set that touches a constant fraction of its edges. We show that such graphs indeed exist.

Beyond Negami's Planar Cover Conjecture

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Joint work with: Marcin Brianksi. James Davies

Negami's famous planar cover conjecture is equivalent to the statement that a connected graph can be embedded in the projective plane if and only if it has a projective planar cover. In 1999, Hliněný proposed extending this conjecture to higher genus non-orientable surfaces. In this talk, we'll put forward a natural extension that encompasses orientable surfaces as well, and discuss some evidence toward it.

Odd Colorings of Graphs

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We consider proper vertex colorings of graphs such that for every vertex v and one (or all) colors that appear on the neighbors of v, appears there an odd number of times. In particular, we discuss some recent results on structural properties (related to sparsity) of graph classes that allow such odd colorings with a bounded number of colors.